UUCMS No.

B.M.S. COLLEGE FOR WOMEN, AUTONOMOUS

BENGALURU – 560004

SEMESTER END EXAMINATION – SEPT/OCT 2023

M.Sc in Mathematics – 2nd Semester

BASIC STATISTICAL METHODS

Course Code: MM206S **QP Code: 12006**
Duration: 3 Hours **Max marks: 70**

Instructions: ***1) All questions carry equal marks.***
2) Answer any five full questions.

1. a) The diameter of an electric cable, say X is assumed to be a continuous random variable with p.d.f.:

$$f(x) = 6x(1-x) \text{ if } 0 \le x \le 1$$

(i) Check that f(x) is p.d.f. and

(ii) Determine a number b such that $(< b) = P(X > b)$

(iii) Compute $P\left(X \le \frac{1}{2} \middle| \frac{1}{3} \le X \le \frac{2}{3}\right)$

b) The following table represents the joint probability distribution of the discrete r.v. $(,Y)$.

X \ Y	1	2
1	0.1	0.2
2	0.1	0.3
3	0.2	0.1

Find
(i) The marginal distributions.
(ii) The conditional distribution of X given $Y = 1$.
(iii) $P(X + Y < 4)$.

(7 + 7)

2. a) Prove for any two random variables X and Y
(i) $E(+Y) = E(X) + E(Y)$ and
(ii) $E() = E()E(Y)$ where X and Y are independent random variables.
b) Find mean and variance of Poisson distribution.

(7 + 7)

3. a) State and prove Chebyshev's inequality.
b) Fit a parabola $y = ax^2 + bx + c$ to the given data

x	10	12	15	23	20
y	14	17	23	25	21

(7 + 7)

4. a) Calculate coefficient of correlation from the following data:

x	9	8	7	6	5	4	3	2	1
y	15	16	14	13	11	12	10	8	9

b) Find the moments of normal distribution and also find the moment generating function of normal distribution.

(7 + 7)

5. a) Three newspapers A, B and C are published in a certain city. It is estimated from a survey that of the adult population: 20% read A, 16% read B, 14% read C, 8% read both A and B, 5% readboth A and C, 4% read both B and C 2% read all three. Find what percentage read at least one of the papers?
b) Define conditional probability. An urn contains 4 red and 7 blue balls. Two balls are drawn one by one without replacement. Find the probability of getting 2 red balls.
c) Prove that for n events $A_1, A_2, \ldots, A_n$, $P(\cap_{i=1}^{n} A_i) \geq \sum_{i=1}^{n} P(A_i) - (n-1)$

(5+ 4+5)

6. a) State and prove Baye's theorem.
b) Define Binomial distribution and find its mean, variance and mgf.

(6+ 8)

7. a) A sample of 900 members has a mean 3.4cms. and std 2.61cms. Is the sample from a large population of mean 3.25cms. and s.d. 2.61cms.? If the population is normal and its mean is unknown, find the 95% and 99% confidence limits.
b) A random sample of size 16 has 53 as mean. The sum of squares of deviations from mean is 135. Can this sample be regarded as taken from the population having 56 as mean ? Also obtain 95% confidence limits for the mean. (at 5% level t value for 4 degree of freedom =2.13).

(7 + 7)

8. a) The following table gives the number of good and bad parts produced by each of the three shifts in a factory:

	Good parts	**Bad parts**	**Total**
Day shift	960	40	1000
Evening shift	940	50	990
Night shift	950	45	995
Total	2850	135	2985

Test whether or not the production of bad parts is independent of the shift on which they were produced.
(χ^2 value at 5% level of significance for 2 degrees of freedom =5.991).
b) In a sample of 8 observations the sum of squares of deviations of the sample values form the sample mean was 94.5 and in another sample of 10 observations it was found to be 101.7. Test whether the difference is significant population at 5% level of significance.
[Given $F_{0.05}(7, 9)$ =3.29].

(7 + 7)

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